



# *p*-hub protection models for reliable hub and spoke network design



## Background

In this project, we develop a new hub location problem named as the '*p*-hub protection models (PHPRO)' as an approach of survivable network design in telecommunications.

## Model Objective

The main goal of the PHPRO models is to build a network which minimizes the total *potential* loss of flows in the face of network failure. As a protective scheme, the model considers *back-up disjoint routes* for interacting flows between OD pairs.

## Model Formulation

$$\text{Minimize } Z = \sum_i \sum_j \sum_k \sum_m \sum_n W_{ij} \left( L_{ijkn}^P X_{ijkn} + \sum_n \overline{L}_{ijkn}^S X_{ijkn} \right)$$

Subject to

$$\sum_k Z_{kk} = p \quad (3 \leq p)$$

$$\sum_k Z_{ik} = 1 \quad \forall i$$

$$Z_{ik} - Z_{kk} \leq 0 \quad \forall i \neq k$$

$$\sum_m X_{ijkm} - Z_{ik} = 0 \quad \forall j > i; k, (i,j,k,m) \in S$$

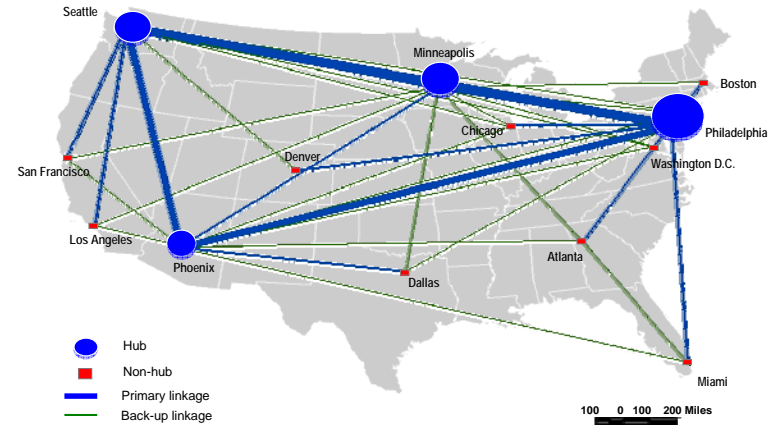
$$\sum_k X_{ijkn} - Z_{jm} = 0 \quad \forall j > i; m, (i,j,k,m) \in S$$

$$X_{ijkn} - Z_{nn} \leq 0 \quad \forall j > i; k, m, n, (i,j,k,m,n) \in S'$$

$$\sum_n X_{ijkn} - X_{ijkn} = 0 \quad \forall j > i; k, m, (i,j,k,m,n) \in S'$$

$$Z_{ik} \in \{0, 1\} \quad \forall i, k$$

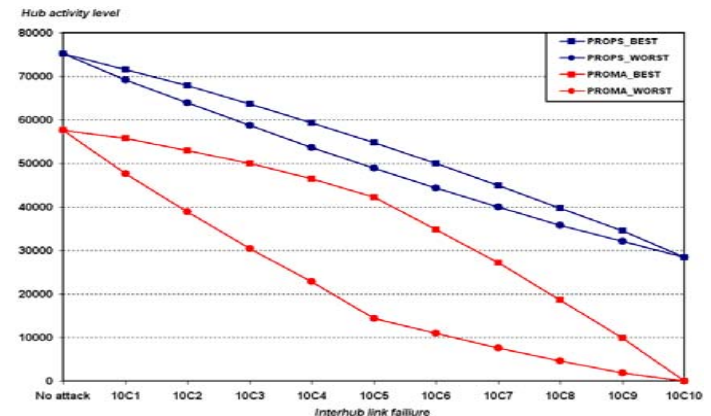
## Optimal Solution for 4-hubs PROPS model



## Evaluation of the model

The model can be compared to the classical hub network model in terms of resiliency to the potential disruption.

Figure below shows that the PROPS model (Blue) is more resilient than the classical hub location model (Red) to the potential attacks on the inter-hub linkages.



Contact: Hyun Kim  
 Center for Urban and Regional Analysis  
<http://cura.osu.edu>  
 Email: [cura@osu.edu](mailto:cura@osu.edu)